Introduction to Mathematics and Modeling

lecture 3

Differentiation

UNIVERSITY OF TWENTE.

academic year : 17-18
lecture : 3
build : November 28, 2017
slides : 47
This week

1. Directives concerning the MLP test
2. Section 3.1: tangents and the derivative at a point
3. Section 3.2: the derivative as a function
4. Section 3.4: velocity
All tests will take place in Therm, using dedicated ‘ChromeBook’ laptops.

- The test will not be available until 8:45 sharp.
- You have 60 minutes to complete the test (75 minutes for dislectic students).
- Be there well in time. If you use public transportation, take one bus or train earlier than usual.
- Although you can start late (but not later than 9:15), this should be an exception. Be well aware that you will disturb your fellow students that already started their test.
- The use of an electronic calculator (or any other device) is not allowed. A calculator will be available on the chromebook as a separate app.
- A trigonometry formula sheet will be issued before the test. This formula sheet can be reviewed on Blackboard.
- A practice test is available.
Tests with MyLabsPlus

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- Elaborate the test questions on paper before you submit an answer to MyLabsPlus.
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- Elaborate the test questions on paper before you submit an answer to MyLabsPlus.
- Put your name and student number on the paper and hand it in after you completed the test.
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The paper is not graded, but can be used as evidence for reviewing purposes.
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The paper is not graded, but can be used as evidence for reviewing purposes.

You can review your test after 12:00.
Elaborate the test questions on paper before you submit an answer to MyLabsPlus.

Put your name and student number on the paper and hand it in after you completed the test.

The paper is not graded, but can be used as evidence for reviewing purposes.

You can review your test after 12:00.

Distribution of points per test:

- test 1: 8 points
- test 2: 12 points
- test 3: 10 points

Your mark is the total amount of points divided by 3, with one-decimal precision, and with minimum 1.
Wrong answer means 0 points!

- Use exact answers, avoid decimals if possible:

\[
0.25 \rightarrow \frac{1}{4}
\]
Wrong answer means 0 points!

- Use exact answers, avoid decimals if possible:
  
  \[
  \frac{1}{4}
  \]

- If decimals are required, use a decimal point:
  
  \[
  3.14 \rightarrow 3.14
  \]
Wrong answer means 0 points!

- Use exact answers, avoid decimals if possible:
  
  0.25 $\rightarrow$ 0.25
  
  0.25 $\rightarrow$ 1/4

- If decimals are required, use a decimal point:
  
  3, 14 $\rightarrow$ 3.14

- Simplify your answers as much as possible:
  
  2/5 $\times$ 1/5 $\rightarrow$ 2/25
  
  2/5 $\rightarrow$ 3/5
  
  3 1/2 $\rightarrow$ 7/2
  
  6x^3/3x^2 $\rightarrow$ 2x
  
  1/2 $\sqrt{4} $ $\rightarrow$ 1
  
  1/3 $\rightarrow$ 1/3 $\sqrt{3}$
Wrong answer means 0 points!

- Use exact answers, avoid decimals if possible:
  
  \[
  0.25 \quad \rightarrow \quad \frac{1}{4}
  \]

- If decimals are required, use a decimal point:
  
  \[
  3.14 \quad \rightarrow \quad 3.14
  \]

- Simplify your answers as much as possible:
  
  \[
  \frac{2}{5} \times \frac{1}{5} \quad \rightarrow \quad \frac{3}{5} \quad \frac{3\frac{1}{2}}{2} \quad \rightarrow \quad \frac{7}{2}
  \]

  \[
  \frac{6x^3}{3x^2} \quad \rightarrow \quad 2x
  \]

  \[
  \frac{1}{2} \sqrt{4} \quad \rightarrow \quad 1 \quad \frac{1}{\sqrt{3}} \quad \rightarrow \quad \frac{1}{3} \sqrt{3}
  \]

- Use the right variable. If \( f \) is a function of \( t \), i.e. \( f(t) = t^2 \), then
  
  \[
  f'(t) = 2t
  \]
The (angle of) inclination is the angle $\theta$ that $\ell$ makes with the $x$-axis.

- The (angle of) inclination is the angle $\theta$ that $\ell$ makes with the $x$-axis.
The (angle of) inclination is the angle \( \theta \) that \( \ell \) makes with the \( x \)-axis.

The angle is measured from the positive \( x \)-axis to \( \ell \).
The (angle of) inclination is the angle $\theta$ that $\ell$ makes with the $x$-axis.

- The angle is measured from the positive $x$-axis to $\ell$.
- Turning counterclockwise means $\theta > 0$. 
The (angle of) inclination is the angle $\theta$ that $\ell$ makes with the $x$-axis. The angle is measured from the positive $x$-axis to $\ell$. Turning counterclockwise means $\theta > 0$. Turning clockwise means $\theta < 0$. 
The slope of a line

The slope of \( \ell \) is defined as \( \tan \theta = \frac{\Delta y}{\Delta x} \).
The slope of a line

- The **slope** of $\ell$ is defined as $\tan \theta = \frac{\Delta y}{\Delta x}$.
- This holds for *every* choice $P_1$ and $P_2$, as long as $P_1 \neq P_2$. 
The slope of a line

The slope of \( \ell \) is \( \tan \theta = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \).
The slope of a line

- The slope of $\ell$ is $\tan \theta = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.
- This holds for every choice $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, as long as $P_1 \neq P_2$. 
Let \( \ell \) be the line through \( P = (x_0, y_0) \) with slope \( m \), then for every point \( (x, y) \neq P \) on \( \ell \) we have

\[
m = \frac{y - y_0}{x - x_0}
\]
Let \( \ell \) be the line through \( P = (x_0, y_0) \) with slope \( m \), then for every point \( (x, y) \neq P \) on \( \ell \) we have

\[
    m = \frac{y - y_0}{x - x_0}
\]

\[
    y - y_0 = m(x - x_0)
\]
Let \( \ell \) be the line through \( P = (x_0, y_0) \) with slope \( m \), then for every point \( (x, y) \neq P \) on \( \ell \) we have

\[
m = \frac{y - y_0}{x - x_0}
\]

\[
y - y_0 = m(x - x_0)
\]

\[
y = m(x - x_0) + y_0.
\]
Let $\ell$ be the line through $P = (x_0, y_0)$ with slope $m$, then for every point $(x, y) \neq P$ on $\ell$ we have

$$m = \frac{y - y_0}{x - x_0} \times (x - x_0)$$

$$y - y_0 = m(x - x_0)$$

$$y = m(x - x_0) + y_0.$$

The **equation of the line through $P$ and with slope $m$** is

$$y = m(x - x_0) + y_0$$
Let $\ell$ be the line through with slope $m$ and with $y$-intercept $b$, then $\ell$ passes through $(0, b)$. 
Let \( \ell \) be the line through with slope \( m \) and with \( y \)-intercept \( b \), then \( \ell \) passes through \((0, b)\).

The equation of \( \ell \) is

\[
y = m(x - 0) + b = mx + b.
\]
Let \( \ell \) be the line through \( P_1 = (x_1, y_1) \) and \( P_2 = (x_2, y_2) \) where \( P_1 \neq P_2 \), then the slope of \( \ell \) is

\[
m = \frac{y_2 - y_1}{x_2 - x_1}.
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Let \( \ell \) be the line through \( P_1 = (x_1, y_1) \) and \( P_2 = (x_2, y_2) \) where \( P_1 \neq P_2 \), then the slope of \( \ell \) is

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m = \frac{y_2 - y_1}{x_2 - x_1}.
\]

The equation of the line through \( P_1 \) and \( P_2 \) is

\[
y = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) + y_1.
\]
Equation of a line passing through two points

- Let \( \ell \) be the line through \( P_1 = (x_1, y_1) \) and \( P_2 = (x_2, y_2) \) where \( P_1 \neq P_2 \), then the slope of \( \ell \) is

\[
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\]

- The equation of the line through \( P_1 \) and \( P_2 \) is

\[
y = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) + y_1.
\]

- Example: the equation of the line through \((1, -1)\) and \((3, 5)\) is

\[
y = \frac{5 - (-1)}{3 - 1} (x - 1) + (-1),
\]
Let \( \ell \) be the line through \( P_1 = (x_1, y_1) \) and \( P_2 = (x_2, y_2) \) where \( P_1 \neq P_2 \), then the slope of \( \ell \) is
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Example: the equation of the line through \((1, -1)\) and \((3, 5)\) is
\[
y = \frac{5 - (-1)}{3 - 1} (x - 1) + (-1),
\]
\[
y = 3(x - 1) - 1,
\]
Let \( \ell \) be the line through \( P_1 = (x_1, y_1) \) and \( P_2 = (x_2, y_2) \) where \( P_1 \neq P_2 \), then the slope of \( \ell \) is
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m = \frac{y_2 - y_1}{x_2 - x_1}.
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The **equation of the line through** \( P_1 \) and \( P_2 \) is
\[
y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1.
\]

**Example:** the equation of the line through \((1, -1)\) and \((3, 5)\) is
\[
y = \frac{5 - (-1)}{3 - 1}(x - 1) + (-1),
\]
\[
y = 3(x - 1) - 1,
\]
\[
y = 3x - 4.
\]
The general equation of a line is
\[ ax + by = c, \]
with \( a \), \( b \) and \( c \) real constants, where \( a \) and \( b \) are not both 0.
The general equation of a line is

\[ ax + by = c, \]

with \( a, b \) and \( c \) real constants, where \( a \) and \( b \) are not both 0.

If \( b \neq 0 \), then

\[ ax + by = c, \]
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\[ by = -ax + c, \]
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If \( b \neq 0 \), then

\[ ax + by = c, \]

\[ by = -ax + c, \]

\[ y = -\frac{a}{b}x + \frac{c}{b}, \]

so the slope is \( -\frac{a}{b} \), and the y-intercept is \( \frac{c}{b} \).
The general equation of a line is

\[ ax + by = c, \]

with \( a, b \) and \( c \) real constants, where \( a \) and \( b \) are not both 0.

- If \( b \neq 0 \), then
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  \[ y = -\frac{a}{b}x + \frac{c}{b}, \]
  so the slope is \( -\frac{a}{b} \), and the \( y \)-intercept is \( \frac{c}{b} \).

- If \( b = 0 \), then \( a \neq 0 \), and \( x = \frac{c}{a} \) is the equation of a **vertical line**.
The horizontal line with $y$-intercept $b$ has slope $0$ and therefore is described by the equation

$$y = b.$$
The horizontal line with \( y \)-intercept \( b \) has slope 0 and therefore is described by the equation

\[ y = b. \]

The vertical line with \( x \)-intercept \( a \) has slope \( \infty \) and is described by the equation

\[ x = a. \]
Assignment: IMM1 - Tutorial 3.1
The derivative of a function $y = f(x)$

We define the **derivative** $f(x)$ at $x_0$ as

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$ 

The number $f'(x_0)$ can be interpreted as:

- the slope of the graph of $y = f(x)$ at the point $(x_0, f(x_0))$;
- the slope of the tangent line to the graph of $y = f(x)$ at the point $(x_0, f(x_0))$;
- the rate of change of $f(x)$ at the point $x_0$. 

Differentiation - Secant.nb
Example: the derivative of $f(x) = x^2$ at 1

For $f(x) = x^2$ we have

<table>
<thead>
<tr>
<th>$h$</th>
<th>$1 + h$</th>
<th>$f(1)$</th>
<th>$f(1 + h)$</th>
<th>$f(1 + h) - f(1)$</th>
<th>$\frac{f(1 + h) - f(1)}{h}$</th>
</tr>
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<tbody>
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<td>1</td>
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<td>0.5</td>
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<td>0.01</td>
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<td>1</td>
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**Example: the derivative of** $f(x) = x^2$ at 1

For $f(x) = x^2$ we have

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<td>2.01</td>
</tr>
<tr>
<td>0.001</td>
<td>1.001</td>
<td>1</td>
<td>1.002001</td>
<td>0.002001</td>
<td>2.001</td>
</tr>
</tbody>
</table>

This suggests: when $h$ approaches 0, then $\frac{f(1+h) - f(1)}{h}$ approaches 2.
Example: the derivative of $f(x) = x^2$ at 1

$$f(1) = 1^2 = 1$$
Example: the derivative of $f(x) = x^2$ at 1

\[
f(1) = 1^2 = 1
\]

\[
\frac{f(1 + h) - f(1)}{h} = \frac{(1 + h)^2 - 1}{h}
\]
Example: the derivative of \( f(x) = x^2 \) at 1

\[
f(1) = 1^2 = 1
\]

\[
\frac{f(1 + h) - f(1)}{h} = \frac{(1 + h)^2 - 1}{h}
\]

\[
= 1 + 2h + h^2 - 1
\]

\[
= \frac{1 + 2h + h^2 - 1}{h}
\]

\[
(x + y)^2 = x^2 + 2xy + y^2
\]
Example: the derivative of \( f(x) = x^2 \) at 1

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f(1) = 1^2 = 1
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\[
\frac{f(1 + h) - f(1)}{h} = \frac{(1 + h)^2 - 1}{h}
\]

\[
= \frac{x + 2h + h^2 - x}{h}
\]

\[
= \frac{2h + h^2}{h}
\]

\[
= 2 + h \left( \lim_{h \to 0} \right)
\]

\[
(x + y)^2 = x^2 + 2xy + y^2
\]
Example: the derivative of $f(x) = x^2$ at 1

$$f(1) = 1^2 = 1$$

$$\frac{f(1 + h) - f(1)}{h} = \frac{(1 + h)^2 - 1}{h}$$

$$= \frac{x + 2h + h^2 - x}{h}$$

$$= \frac{2h + h^2}{h}$$

$$(x + y)^2 = x^2 + 2xy + y^2$$
Example: the derivative of $f(x) = x^2$ at 1

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$$= \frac{2h + h^2}{h}$$

$$= 2 + h$$

$(x + y)^2 = x^2 + 2xy + y^2$
Example: the derivative of $f(x) = x^2$ at 1

$$f(1) = 1^2 = 1$$

$$\frac{f(1 + h) - f(1)}{h} = \frac{(1 + h)^2 - 1}{h}$$

$$= \frac{1 + 2h + h^2 - 1}{h}$$

$$= \frac{2h + h^2}{h}$$

$$= 2 + h$$

$$f'(1) = \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} = 2.$$
Example: the tangent line of $f(x) = x^2$ at $(1, 1)$

The tangent line has slope $f'(1) = 2$ and passes through $(1, f(1)) = (1, 1)$, hence the tangent line is described by the equation $y = 2 \cdot (x - 1) + 1 = 2x - 1$. 

Example: the derivative of $f(x) = x^2$ at $a$

$$f(x) = x^2.$$
Example: the derivative of $f(x) = x^2$ at $a$

\[
f(x) = x^2.
\]

\[
\frac{f(a + h) - f(a)}{h} = \frac{(a + h)^2 - a^2}{h}
\]
Example: the derivative of \( f(x) = x^2 \) at \( a \)

\[
f(x) = x^2.
\]

\[
\frac{f(a + h) - f(a)}{h} = \frac{(a + h)^2 - a^2}{h} = \frac{a^2 + 2ah + h^2 - a^2}{h} = \frac{2ah + h^2}{h} = 2a + h.
\]

\((x + y)^2 = x^2 + 2xy + y^2\)
Example: the derivative of \( f(x) = x^2 \) at \( a \)

\[
f(x) = x^2.
\]

\[
\frac{f(a + h) - f(a)}{h} = \frac{(a + h)^2 - a^2}{h}
\]

\[
= \frac{a^2 + 2ah + h^2 - a^2}{h}
\]

\[
= \frac{2ah + h^2}{h}
\]

\[
= 2a + h.
\]

\[
\frac{f(a + h) - f(a)}{h} = \lim_{h \to 0} \frac{(a + h)^2 - a^2}{h} = 2a.
\]

\[
(x + y)^2 = x^2 + 2xy + y^2
\]
Example: the derivative of $f(x) = x^2$ at $a$

$$f(x) = x^2.$$ 

$$f(a + h) - f(a) = (a + h)^2 - a^2$$

$$= (a^2 + 2ah + h^2) - a^2$$

$$= 2ah + h^2$$
Example: the derivative of $f(x) = x^2$ at $a$

$$f(x) = x^2.$$ 

$$\frac{f(a + h) - f(a)}{h} = \frac{(a + h)^2 - a^2}{h}$$

$$= \frac{a^2 + 2ah + h^2 - a^2}{h}$$

$$= \frac{2ah + h^2}{h}$$

$$= 2a + h.$$
Example: the derivative of $f(x) = x^2$ at $a$

$$f(x) = x^2.$$

$$\frac{f(a + h) - f(a)}{h} = \frac{(a + h)^2 - a^2}{h} = \frac{a^2 + 2ah + h^2 - a^2}{h} = \frac{2ah + h^2}{h} = 2a + h.$$

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = 2a.$$
Example: the derivative of $f(x) = \sqrt{x}$ at $a$

$$f(a) = \sqrt{a}.$$
Example: the derivative of \( f(x) = \sqrt{x} \) at \( a \)

\[
\begin{align*}
  f(a) &= \sqrt{a}.
  \\
  \frac{f(a + h) - f(a)}{h} &= \frac{\sqrt{a + h} - \sqrt{a}}{h}
\end{align*}
\]
Example: the derivative of $f(x) = \sqrt{x}$ at $a$

$$f(a) = \sqrt{a}.$$ 

$$\frac{f(a + h) - f(a)}{h} = \frac{\sqrt{a + h} - \sqrt{a}}{h}$$

$$= \frac{(\sqrt{a + h} - \sqrt{a})(\sqrt{a + h} + \sqrt{a})}{h(\sqrt{a + h} + \sqrt{a})}$$

$$= \frac{1}{2\sqrt{a}}$$
Example: the derivative of \( f(x) = \sqrt{x} \) at \( a \)

\[
\begin{align*}
f(a) &= \sqrt{a}.
\end{align*}
\]

\[
\begin{align*}
\frac{f(a + h) - f(a)}{h} &= \frac{\sqrt{a + h} - \sqrt{a}}{h} \\
&= \frac{(\sqrt{a + h} - \sqrt{a})(\sqrt{a + h} + \sqrt{a})}{h(\sqrt{a + h} + \sqrt{a})} \\
&= \frac{(a + h) - a}{h(\sqrt{a + h} + \sqrt{a})} \\
&= \frac{1}{2\sqrt{a + h + \sqrt{a}}}
\end{align*}
\]
Example: the derivative of \( f(x) = \sqrt{x} \) at \( a \)

\[
\begin{align*}
f(a) &= \sqrt{a}.

\frac{f(a + h) - f(a)}{h} &= \frac{\sqrt{a + h} - \sqrt{a}}{h}

&= \frac{(\sqrt{a + h} - \sqrt{a})(\sqrt{a + h} + \sqrt{a})}{h(\sqrt{a + h} + \sqrt{a})}

&= \frac{(\alpha + h) - \alpha}{h(\sqrt{a + h} + \sqrt{a})}

&= \frac{1}{2\sqrt{a}}.
\end{align*}
\]
Example: the derivative of $f(x) = \sqrt{x}$ at $a$

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$$\frac{f(a + h) - f(a)}{h} = \frac{\sqrt{a + h} - \sqrt{a}}{h}$$

$$= \frac{(\sqrt{a + h} - \sqrt{a})(\sqrt{a + h} + \sqrt{a})}{h(\sqrt{a + h} + \sqrt{a})}$$

$$= \frac{(a + h) - a}{h(\sqrt{a + h} + \sqrt{a})}$$

$$= \frac{h}{h(\sqrt{a + h} + \sqrt{a})}$$

$$= \frac{1}{2\sqrt{a + h} + \sqrt{a}}.$$
Example: the derivative of \( f(x) = \sqrt{x} \) at \( a \)

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f(a) = \sqrt{a}.
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\frac{f(a + h) - f(a)}{h} = \frac{\sqrt{a + h} - \sqrt{a}}{h}
\]

\[
= \frac{(\sqrt{a + h} - \sqrt{a})(\sqrt{a + h} + \sqrt{a})}{h(\sqrt{a + h} + \sqrt{a})}
\]

\[
= \frac{(a + h) - a}{h(\sqrt{a + h} + \sqrt{a})}
\]

\[
= \frac{h}{h(\sqrt{a + h} + \sqrt{a})}
\]

\[
= \frac{1}{\sqrt{a + h} + \sqrt{a}}.
\]

\((x + y)(x - y) = x^2 - y^2\)
Example: the derivative of $f(x) = \sqrt{x}$ at $a$

\[
f(a) = \sqrt{a}.
\]

\[
\frac{f(a + h) - f(a)}{h} = \frac{\sqrt{a + h} - \sqrt{a}}{h}
\]

\[
= \frac{(\sqrt{a + h} - \sqrt{a})(\sqrt{a + h} + \sqrt{a})}{h(\sqrt{a + h} + \sqrt{a})}
\]

\[
= \frac{(a + h) - a}{h(\sqrt{a + h} + \sqrt{a})}
\]

\[
= \frac{h}{h(\sqrt{a + h} + \sqrt{a})}
\]

\[
= \frac{1}{\sqrt{a + h} + \sqrt{a}}.
\]

\[
f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \frac{1}{2\sqrt{a}}.
\]

$(x + y)(x - y) = x^2 - y^2$
Example: the derivative of $f(x) = 1/x$ at $a \neq 0$

$$f(a) = \frac{1}{a} \quad (a \neq 0).$$
Example: the derivative of \( f(x) = 1/x \) at \( a \neq 0 \)

\[
f(a) = \frac{1}{a} \quad (a \neq 0).
\]

\[
\frac{f(a + h) - f(a)}{h} = \frac{1}{a + h} - \frac{1}{a} = \frac{1}{h} \left( \frac{1}{a + h} - \frac{1}{a} \right)
\]
Example: the derivative of \( f(x) = 1/x \) at \( a \neq 0 \)

\[
f(a) = \frac{1}{a} \quad (a \neq 0).
\]

\[
\frac{f(a + h) - f(a)}{h} = \frac{1}{a + h} - \frac{1}{a} = \frac{1}{h} \left( \frac{1}{a + h} - \frac{1}{a} \right)
\]

\[
= \frac{1}{h} \left( \frac{a}{a(a + h)} - \frac{a + h}{a(a + h)} \right)
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Example: the derivative of \( f(x) = 1/x \) at \( a \neq 0 \)

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\frac{f(a + h) - f(a)}{h} = \frac{1}{a + h} - \frac{1}{a} = \frac{1}{h} \left( \frac{1}{a + h} - \frac{1}{a} \right)
\]

\[
= \frac{1}{h} \left( \frac{a}{a(a + h)} - \frac{a + h}{a(a + h)} \right)
\]

\[
= \frac{1}{h} \left( \frac{a - (a + h)}{a(a + h)} \right)
\]
Example: the derivative of $f(x) = 1/x$ at $a \neq 0$

\[ f(a) = \frac{1}{a} \quad (a \neq 0). \]

\[
\frac{f(a + h) - f(a)}{h} = \frac{1}{a + h} - \frac{1}{a} = \frac{1}{h} \left( \frac{1}{a + h} - \frac{1}{a} \right)
\]

\[ = \frac{1}{h} \left( \frac{a}{a(a + h)} - \frac{a + h}{a(a + h)} \right) \]

\[ = \frac{1}{h} \left( \frac{a - (a + h)}{a(a + h)} \right) = \frac{1}{h} \left( \frac{-h}{a(a + h)} \right) \]
Example: the derivative of $f(x) = 1/x$ at $a \neq 0$

$$f(a) = \frac{1}{a} \quad (a \neq 0).$$

$$f(a + h) - f(a) = \frac{1}{a + h} - \frac{1}{a} = \frac{1}{h} \left( \frac{1}{a + h} - \frac{1}{a} \right)$$

$$= \frac{1}{h} \left( \frac{a}{a(a + h)} - \frac{a + h}{a(a + h)} \right)$$

$$= \frac{1}{h} \left( \frac{a - (a + h)}{a(a + h)} \right) = \frac{1}{h} \left( \frac{-h}{a(a + h)} \right)$$

$$= -\frac{1}{a(a + h)}.$$
Example: the derivative of $f(x) = 1/x$ at $a \neq 0$

$$f(a) = \frac{1}{a} \quad (a \neq 0).$$

$$\frac{f(a + h) - f(a)}{h} = \frac{\frac{1}{a + h} - \frac{1}{a}}{h} = \frac{1}{h} \left( \frac{1}{a + h} - \frac{1}{a} \right)$$

$$= \frac{1}{h} \left( a - (a + h) \right) = \frac{1}{h} \left( \frac{-h}{a(a + h)} \right)$$

$$= -\frac{1}{a(a + h)}.\)$$

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \frac{-1}{a^2}.\)$$
(1) Show that \( b^3 - a^3 = (b - a)(b^2 + ab + a^2) \).

(2) Let \( f(x) = x^3 \). Use the definition of the derivative to show that for all real numbers \( a \) the following holds:

\[
f'(a) = 3a^2.
\]

Assignment: IMM1 - Tutorial 3.2
The derivative of the function $f$ is the function $f'$ whose value at $x$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.$$

- The function $f$ is **differentiable at** $x$ if $f'(x)$ exists.
The derivative of the function $f$ is the function $f'$ whose value at $x$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.$$ 

- The function $f$ is differentiable at $x$ if $f'(x)$ exists.
- The process of calculating $f'$ is called **differentiation**.
The derivative as a function

The **derivative of the function** $f$ is the function $f'$ whose value at $x$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.$$  

- The function $f$ is **differentiable at** $x$ if $f'(x)$ exists.
- The process of calculating $f'$ is called **differentiation**.
- Alternative notations for the derivative are

$$\frac{df}{dx}$$

and

$$\frac{d}{dx} f(x).$$
Example: the derivative of $f(x) = x^2$

We already evaluated the derivative of $f$ at $a$:

$$f'(a) = 2a.$$  

Replace $a$ by $x$: the derivative of $f$ is the function $f'(x) = 2x$. 

$f(x) = x^2$, $f'(x) = 2x$
Example: the derivative of \( f(x) = \sqrt{x} \)

\[ f(x) = \sqrt{x} \]
\[ f'(x) = \frac{1}{2\sqrt{x}} \]

The derivative of \( f \) at \( a \) is \( f'(a) = \frac{1}{2\sqrt{a}} \).

Replace \( a \) by \( x \): the derivative of \( f \) is the function \( f'(x) = \frac{1}{2\sqrt{x}} \) \((x > 0)\).
Example: the derivative of $f(x) = 1/x$

The derivative of $f$ at $a$ is $f'(a) = -\frac{1}{a^2}$.

Replace $a$ by $x$: the derivative of $f$ is the function $f'(x) = -\frac{1}{x^2}$ ($x \neq 0$).
The graph of \( f(x) = \begin{cases} x + 1 & \text{if } x \geq 0, \\ -x & \text{if } x < 0 \end{cases} \) does not have a derivative at \( x = 0 \).

- A derivative does not exist at a point where the graph is discontinuous.
The graph of $y = f(x) = |x|$ does not have a derivative at $x = 0$.

- A derivative does not exist at a point where the graph has a sharp spike (called a salient).
Warning: derivatives are not always defined!

The graph of \( y = f(x) = \sqrt[3]{x} \) does not have a derivative at \( x = 0 \).

- A derivative does not exist at a point where the graph has a vertical tangent.
The function \( f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases} \) is differentiable at 0.

- Piecewise defined functions do not always pose problems.
Recursion Formula

For all \( n \geq 1 \) we have

\[
\frac{d}{dx}(x^{n+1}) = x \frac{d}{dx}(x^n) + x^n
\]
Recursion Formula

For all $n \geq 1$ we have

$$\frac{d}{dx}(x^{n+1}) = x \frac{d}{dx}(x^n) + x^n$$

$$\frac{d}{dx}(x^{n+1}) = \lim_{h \to 0} \frac{(x + h)^{n+1} - x^{n+1}}{h}$$
Integer powers of $x$

Recursion Formula

For all $n \geq 1$ we have

$$\frac{d}{dx}(x^{n+1}) = x \frac{d}{dx}(x^n) + x^n$$

$$\frac{d}{dx}(x^{n+1}) = \lim_{h \to 0} \frac{(x + h)^{n+1} - x^{n+1}}{h}$$

$$= \lim_{h \to 0} \frac{(x + h)(x + h)^n - x \cdot x^n}{h}$$
Integer powers of $x$

**Recursion Formula**

For all $n \geq 1$ we have

$$\frac{d}{dx}(x^{n+1}) = x \frac{d}{dx}(x^n) + x^n$$

\[\begin{align*}
\frac{d}{dx}(x^{n+1}) &= \lim_{h \to 0} \frac{(x + h)^{n+1} - x^{n+1}}{h} \\
&= \lim_{h \to 0} \frac{(x + h)(x + h)^n - x \cdot x^n}{h} \\
&= \lim_{h \to 0} \frac{x(x + h)^n - x \cdot x^n + h(x + h)^n}{h}
\end{align*}\]
Recursion Formula

For all \( n \geq 1 \) we have

\[
\frac{d}{dx}(x^{n+1}) = x \frac{d}{dx}(x^n) + x^n
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Recursion Formula

For all $n \geq 1$ we have

$$\frac{d}{dx}(x^{n+1}) = x \frac{d}{dx}(x^n) + x^n$$

\[
\begin{align*}
\frac{d}{dx}(x^{n+1}) &= \lim_{h \to 0} \frac{(x + h)^{n+1} - x^{n+1}}{h} \\
&= \lim_{h \to 0} \frac{(x + h)(x + h)^n - x \cdot x^n}{h} \\
&= \lim_{h \to 0} \frac{x(x + h)^n - x \cdot x^n + h(x + h)^n}{h} \\
&= \lim_{h \to 0} x \frac{(x + h)^n - x^n}{h} + \lim_{h \to 0} (x + h)^n \\
&= x \frac{d}{dx}(x^n) + x^n.
\end{align*}
\]
Integer powers of $x$

Let $n = 1$: \[ \frac{d}{dx}(x^2) = x \frac{d}{dx}(x) + x = x \cdot 1 + x = 2x. \]
Integer powers of $x$

- Let $n = 1$: \[ \frac{d}{dx}(x^2) = x \frac{d}{dx}(x) + x = x \cdot 1 + x = 2x. \]

- Let $n = 2$: \[ \frac{d}{dx}(x^3) = x \frac{d}{dx}(x^2) + x^2 = x \cdot 2x + x^2 = 3x^2. \]
Let $n = 1$: \[
\frac{d}{dx} (x^2) = x \frac{d}{dx} (x) + x = x \cdot 1 + x = 2x.
\]

Let $n = 2$: \[
\frac{d}{dx} (x^3) = x \frac{d}{dx} (x^2) + x^2 = x \cdot 2x + x^2 = 3x^2.
\]

Let $n = 3$: \[
\frac{d}{dx} (x^4) = x \frac{d}{dx} (x^3) + x^3 = x \cdot 3x^2 + x^3 = 4x^3.
\]
Let $n = 1$: \[ \frac{d}{dx}(x^2) = x \frac{d}{dx}(x) + x = x \cdot 1 + x = 2x. \]

Let $n = 2$: \[ \frac{d}{dx}(x^3) = x \frac{d}{dx}(x^2) + x^2 = x \cdot 2x + x^2 = 3x^2. \]

Let $n = 3$: \[ \frac{d}{dx}(x^4) = x \frac{d}{dx}(x^3) + x^3 = x \cdot 3x^2 + x^3 = 4x^3. \]

Let $n = 4$: \[ \frac{d}{dx}(x^5) = \ldots \]

Let $n = 5$: \[ \frac{d}{dx}(x^6) = \ldots \]
Integer powers of $x$

- Let $n = 1$: \[ \frac{d}{dx}(x^2) = x \frac{d}{dx}(x) + x = x \cdot 1 + x = 2x. \]

- Let $n = 2$: \[ \frac{d}{dx}(x^3) = x \frac{d}{dx}(x^2) + x^2 = x \cdot 2x + x^2 = 3x^2. \]

- Let $n = 3$: \[ \frac{d}{dx}(x^4) = x \frac{d}{dx}(x^3) + x^3 = x \cdot 3x^2 + x^3 = 4x^3. \]

- Let $n = 4$: \[ \frac{d}{dx}(x^5) = \ldots \]

- Let $n = 5$: \[ \frac{d}{dx}(x^6) = \ldots \]
Integer powers of $x$

- Let $n = 1$: \( \frac{d}{dx}(x^2) = x \frac{d}{dx}(x) + x = x \cdot 1 + x = 2x. \)

- Let $n = 2$: \( \frac{d}{dx}(x^3) = x \frac{d}{dx}(x^2) + x^2 = x \cdot 2x + x^2 = 3x^2. \)

- Let $n = 3$: \( \frac{d}{dx}(x^4) = x \frac{d}{dx}(x^3) + x^3 = x \cdot 3x^2 + x^3 = 4x^3. \)

- Let $n = 4$: \( \frac{d}{dx}(x^5) = \ldots \)

- Let $n = 5$: \( \frac{d}{dx}(x^6) = \ldots \)

**Theorem**

For all $n \geq 1$ we have \( \frac{d}{dx}(x^n) = n x^{n-1} \)
**Theorem**

For all real numbers $\alpha$ we have

$$\frac{d}{dx}(x^\alpha) = \alpha x^{\alpha-1}$$
Powers of $x$

**Theorem**

*For all real numbers $\alpha$ we have*

$$\frac{d}{dx}(x^\alpha) = \alpha x^{\alpha-1}$$

**Check:**

- Let $\alpha = \frac{1}{2}$, then

$$\frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} x^{\frac{1}{2}-1}.$$
Theorem

For all real numbers $\alpha$ we have

$$\frac{d}{dx}(x^{\alpha}) = \alpha x^{\alpha-1}$$

Check:

- Let $\alpha = \frac{1}{2}$, then

$$\frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} x^{\frac{1}{2}-1}.$$

- Let $\alpha = -1$, then

$$\frac{d}{dx}(x^{-1}) = \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2} = -x^{-2} = -1 x^{(-1)-1}.$$
Exercises

- Define \( f(x) = mx + b \). Find the derivative \( f' \) and draw a plot containing the graphs of \( f \) and \( f' \) for the following values of \( m \) and \( b \):

  (1) \( m = 0.5, \ b = 1 \),
  (2) \( m = 1, \ b = 0 \),
  (3) \( m = 2, \ b = -1 \).

Assignment: IMM1 - Tutorial 3.3
Consider a moving object and assume that we know the traveled distance as a function of time $s(t)$.

- If the object moves from $s(t_A)$ to $s(t_B)$, the **displacement** is $s(t_B) - s(t_A)$.

- The **average velocity over the interval** $(t_A, t_B)$ is the displacement per elapsed time.

$$\frac{s(t_B) - s(t_A)}{t_B - t_A}.$$
Consider a moving object and assume that we know the traveled distance as a function of time $s(t)$.

- The **velocity at time** $t_A$ is the limit of the average velocity over the interval $(t_A, t_B)$ where $t_B$ approaches $t_A$:

$$v(t_A) = \lim_{t_B \to t_A} \frac{s(t_B) - s(t_A)}{t_B - t_A}.$$
Consider a moving object and assume that we know the traveled distance as a function of time $s(t)$.

- The **velocity at time** $t_A$ is the limit of the average velocity over the interval $(t_A, t_B)$ where $t_B$ approaches $t_A$:

$$v(t_A) = \lim_{t_B \to t_A} \frac{s(t_B) - s(t_A)}{t_B - t_A}.$$
Consider a moving object and assume that we know the traveled distance as a function of time $s(t)$.

- **The velocity at time** $t_A$ is the limit of the average velocity over the interval $(t_A, t_B)$ where $t_B$ approaches $t_A$:

$$v(t_A) = \lim_{t_B \to t_A} \frac{s(t_B) - s(t_A)}{t_B - t_A}.$$
Velocity

\[ v(t_A) = \lim_{t_B \to t_A} \frac{s(t_B) - s(t_A)}{t_B - t_A}. \]

Define \( h = t_B - t_A \), then

- \( t_B = t_A + h \) and
- “\( t_B \to t_A \)” is equivalent to “\( h \to 0 \).”

\[ v(t_A) = \lim_{t_B \to t_A} \frac{s(t_B) - s(t_A)}{t_B - t_A} = \lim_{h \to 0} \frac{s(t_A + h) - s(t_A)}{h} = s'(t_A). \]
Velocity

\[ v(t_A) = \lim_{t_B \to t_A} \frac{s(t_B) - s(t_A)}{t_B - t_A}. \]

Define \( h = t_B - t_A \), then

- \( t_B = t_A + h \) and
- \( "t_B \to t_A" \) is equivalent to \( "h \to 0" \).

\[ v(t_A) = \lim_{t_B \to t_A} \frac{s(t_B) - s(t_A)}{t_B - t_A} = \lim_{h \to 0} \frac{s(t_A + h) - s(t_A)}{h} = s'(t_A). \]

Velocity is the derivative of displacement.
Example: the motion of a rocket

- **Liftoff**
- **Climb Phase**
- **Apogee & Ejection**
- **Recovery System Deploys**
- **Landing**
Example: the motion of a rocket

Question: when did the rocket reach its highest point (apex)?
Example: the motion of a rocket

Question: when did the rocket reach its highest point (apex)?

Answer: at $t \approx 8$ seconds.
Example: the motion of a rocket

Question: for how many seconds did the engine burn?
Example: the motion of a rocket

Question: for how many seconds did the engine burn?

Answer: 2 seconds.
Example: the motion of a rocket

\[ v(t) \] (ft/sec)

-100  -50   0    50    100    150    200

\[ t \] (sec)

2  4  6  8  10  12  14  16  18  20  22  24  26

Question: when did the parachute open?

Answer: at \[ t = 10 \] seconds.
Example: the motion of a rocket

Question: when did the parachute open?

Answer: at $t = 10$ seconds.
Example: the motion of a rocket

Question: what happens here?
Example: the motion of a rocket

Question: what happens here?

Answer: after approximately 12 seconds the rocket reaches terminal velocity, which it keeps for about 8 seconds.
Example: the motion of a rocket

Question: what happens here?
Example: the motion of a rocket

Question: what happens here?

Answer: the rocket hits the ground at $t \approx 20$ seconds.
Example: the motion of a rocket

Question: what is the physical interpretation of the second derivative?
Example: the motion of a rocket

\[ t \quad (\text{sec}) \]

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</table>

Question: what is the physical interpretation of the second derivative?

Answer: acceleration
Which is better: falling or crashing?

The acceleration of gravity is \( g = 9.81 \, \text{m/s}^2 \).

The velocity at time \( t \) is \( v(t) = gt \, \text{m/s} \).

The distance travelled is \( s(t) = \frac{1}{2}gt^2 \, \text{m} \).

The fall time \( t_0 \) is found by solving \( s(t_0) = 10 \Rightarrow t_0 = 1.43 \, \text{sec} \).

The velocity when hitting the ground is \( v(t_0) = 14.0 \, \text{m/s} \approx 50.4 \, \text{km/h} \).

50 km/h
Which is better: falling or crashing?

- The acceleration of gravity is \( g = 9.81 \text{ m/s}^2 \).
Which is better: falling or crashing?

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- The fall time $t_0$ is found by solving $s(t_0) = 10 \Rightarrow t_0 = 1.43 \text{ sec}$.
- The velocity when hitting the ground is
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  v(t_0) = 14.0 \text{ m/s} = \frac{14.0}{1000} \times 3600 \approx 50.4 \text{ km/h}.
  \]
Physical principles

(1) **In a capacitor, the charge** \( Q \) **on the plates is proportional to the voltage** \( V \) **over the plates: hence** \( Q = CV \), **where** \( C \) **is the capacity**.

(2) **The current through a lead is the amount of charge per second flowing through the lead.**
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- After taking the limit \(\Delta t \to 0\) we see that \(I(t) = CV'(t)\)
Assignment: IMM1 - Tutorial 3.4